LAB REPORT: LAB 1

TNM079, MODELING AND ANIMATION

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Abstract

This is the first lab in a series of six labs in the course TNM079 Modeling and Animation at Linköping University. This report studies the half-edge mesh data structure, as well as physical attributes and basic mesh calculations such as curvature, vertex normals, mesh area and mesh volume, among other methods to compute the mesh structure. The result shows that although the half-edge data structure is a bit complex to implement and requires more memory than a simple mesh structure, it is efficient in performing mesh operations that require access to adjacent vertices.

1 Introduction

In computer graphics, meshes can be stored in a numerous different ways, and there are many different kinds of mesh data structures. A common representation of a polygon mesh is a shared list of vertices and a list of faces storing pointers for its vertices. This method is efficient and convenient for various purposes, but proves ineffective in some domains. For example, there is no simple way to access neighbouring triangles, contributing to its ineffectiveness. Therefore, the half-edge data structure is introduced. This data structure is a performance effective method to store mesh data, at the cost of more memory. In this report it is assumed that the meshes used are of the closed manifold triangle type.

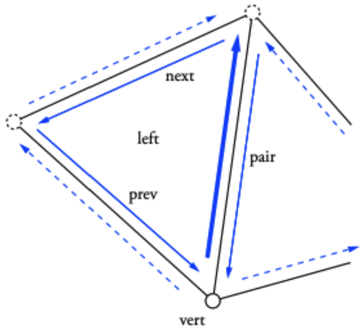


Figure 1: The half-edge data structure as seen from the bold half-edge.

2 Background

2.1 Implement the half-edge mesh and neighbour access

A triangle mesh structure uses a vertex list and a face (triangle) list to define the topology and geometry of the mesh. For a half-edge mesh, the same information is stored as well as information about the edges. This enables access to neighbouring faces. Every edge is ”split” down its length, therefore, only information about the *left* face is stored explicitly. To access information about the *right* face, one has to go through the *pair* of the half-edge. The information accessed also includes *next* and *prev* pointers, which can be used to traverse the half-edges of the left face in a counter-clockwise fashion. Lastly, the information includes a pointer to a vertex used to access the half-edge called *vert*. With all this information, given a vertex, the neighbouring topology can be accessed through the half-edge structure described above (see Figure 1) [1].

2.2 Calculate vertex normals

The normal vector will point perpendicular to the local surface and is a simple geometric differential. Given the three vertices $\mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{v}_3$ and their counter clockwise orientation, a plane normal can be constructed with the cross product of $(\mathbf{v}_2 - \mathbf{v}_1)$ and $(\mathbf{v}_3 - \mathbf{v}_1)$. This will form a normal vector with the desired properties (see Equation 1).

$$\mathbf{n} = (\mathbf{v}_2 - \mathbf{v}_1) \times (\mathbf{v}_3 - \mathbf{v}_1) \tag{1}$$

However, this computation can cause linear properties in the mesh. Therefore, the technique mean weighted equally (MWE) can be implemented. MWE is defined as the normalized sum of the adjacent face normals (see Equation 2).

$$\mathbf{n}_{v_i} = \widehat{\sum_{j \in N_1(i)}^{n} \mathbf{n}_{f_j}} \tag{2}$$

$N_1(i)$ is all the faces sharing vertex v_i, and is called the 1-ring neighbourhood.

2.3 Calculate surface area of a mesh

With the use of a Riemann sum an integral can be approximated discreetly, which can be implemented and used when calculating the area of a mesh. It follows that the area of a mesh is the sum of the areas of each individual face. In Equation 3, $A(f_i$ is the surface area of the i:th face. By calculating half the magnitude of the cross product between any two edges of the triangle, the area of a triangle face can be calculated.

$$A_S = \int_S dA \approx \sum_{i \in S} A(f_i) \tag{3}$$

2.4 Calculate volume of a mesh

An analytical computation of the enclosed volume of the closed manifold mesh is not possible, but the volume can be sufficiently approximated through the divergence theorem (alt. Gauss' Theorem). It relates the volume and surface integrals (see Equation 4.

$$\int_S \mathbf{F} \cdot \mathbf{n} dA = \int_V \nabla \cdot \mathbf{F} d\tau \tag{4}$$

In the theorem, the relation between the surface integral of a vector field times the unit normal equals the volume integral of the divergence of the same vector field. A vector field with special properties can be selected since this is true for any vector field. If it is assumed that $\mathbf{F}$ has constant divergence ($\nabla \cdot \mathbf{F} = c$), the volume integral becomes

$$\int_V \nabla \cdot \mathbf{F} d\tau = \int_V c d\tau = c \int_V d\tau = cV \tag{5}$$

The divergence can be calculated with $\mathbf{F} = \overline{\mathbf{F}} = (x, y, z)$, which leads to:

$$\begin{aligned} &\nabla \cdot \overline{\mathbf{F}} \\ &\quad = \nabla \cdot (x, y, z) \\ &\quad = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}) \cdot (x, y, z) \\ &\quad = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} \\ &\quad = 3 \end{aligned} \tag{6}$$

From this a discreet formula can be acquired. The volume integral will compute $3V$, and the volume can be approximated according to Equation 7, where $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ are the vertices of the i:th face.

$$3V = \sum_{i \in S} \frac{(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3) f_i}{3} \cdot \mathbf{n}(f_i) A(f_i) \tag{7}$$

2.5 Implement and visualize Gaussian curvature

Curvature can be seen as one of the most important mesh quality measures. It describes the smoothness of the surface, as well as how the normal at point p changes as we move it along the surface. To determine the curvature, the schemes *Gaussian curvature* or *mean curvature* can be used. In this lab the gaussian curvature was implemented, defined as $K = k_1 k_2$. Equation 8 an be used to implement the curvature.

$$K = \frac{1}{A}(2\pi - \sum_{j \in N_1(i)} \theta_j) \tag{8}$$

The principal curvatures are defined as the maximal and minimal curvatures passing through any given point on the mesh surface. K is calculated by Equation 8, where A is the calculated area of the 1-ring neighbourhood.

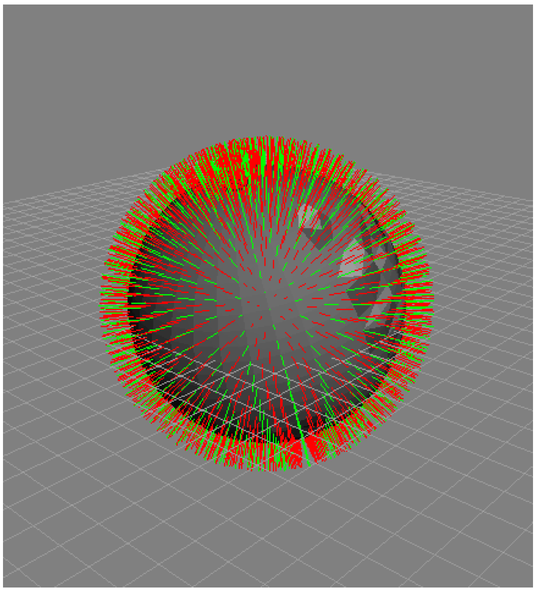


Figure 2: Visualization of vertex normals.

3 Results

In this section detailed information is provided regarding the output of the lab.

3.1 Generating the vertex normals

In Figure 2 the vertex normals are visualized as green lines and face normals as red lines.

The normals are calculated with the cross product as previously mentioned in Section 2.2 (see Equation 1).

3.2 Surface area of the mesh

By studying spheres with radii 0.1, 0.5 and 1 the analytically correct areas can be calculated and compared to the mesh representations of the same spheres. Table 3.2 displays the comparisons. The calculated area is smaller than the actual area because its an approximation of the area. The sphere is not completely smooth, which leads to the sphere having a smaller area.

Table 1: Comparison of area values for spheres.

Sphere radius	Actual A	Calculated A
0.1	0.12566	0.12511
0.5	3.14159	3.12775
1.0	12.5664	12.5110

3.3 Volume of the mesh

For the volume calculation of the mesh, the same spheres can be studied similarly to Section 3.2. The analytically correct volumes can be calculated and compared to the calculation from the mesh representation. See Table 3.3 for comparisons. The calculated volume is smaller than the actual volume because its simply an approximation of the volume. The sphere is not completely smooth and therefore lacks in volume. The used meshes are only approximations with a finite number of vertices. If the number of vertices would increase, the approximation would become more accurate.

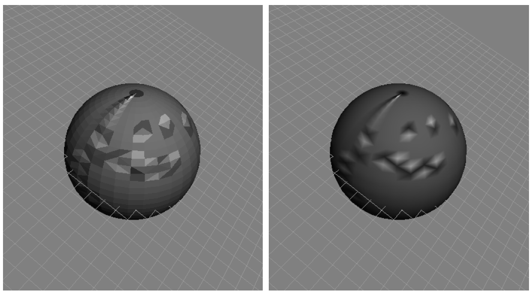
Table 2: Comparison of volume values for spheres.

Sphere radius	Actual V	Calculated V
0.1	0.0041888	0.0041519
0.5	0.523360	0.518988
1.0	4.18879	4.15190

3.4 Curvature calculations

As can be noticed in Figure 3, the Gaussian curvature has some issues with specific areas of the mesh. This results from the calculated curvature becoming higher the denser the triangles are, as can be seen at the top of the sphere where there is a black circle.

Analytically, there should be the same color across the surface since the curvature is inversely proportional to the radius of the sphere. As can be seen in Figure 3 this is not the case. The different ways the triangles are placed will affect the curvature, and the Gaussian curvature is easily swayed since it multiplies principal curvatures k_1 and k_2.



(a) (b)

Figure 3: Result from calculations of the mesh showing (a) vertex curvature compared to face curvature in (b).

4 Lab partner and grade

My lab partner was Viktor Tholén, and I am aiming for grade 3.

References

[1] Emma Broman Mark Eric Dieckmann, Robin Skånberg. *Mesh Data Structures*. Modeling and animation, lab 1, 2021.